Trading in Markets with Noisy Information: An Evolutionary Analysis

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ABSTRACT
We analyse the value of information in a stock market where information can be noisy and costly, using techniques from empirical game theory. Previous work has shown that the value of information follows a J-curve, where averagely informed traders perform below market average, and only insiders prevail. Here we show that both noise and cost can change this picture, in several cases leading to opposite results where insiders perform below market average, and averagely informed traders prevail. These results provide insight into the complexity of real marketplaces, and show under which conditions a broad mix of different trading strategies might be sustainable.

1. INTRODUCTION
Markets play a central role in today’s society and find wide application ranging from stock markets to consumer-to-consumer e-commerce [2, 3]. Success in market trading will greatly depend on traders having accurate market forecasts. There are two main types of trading strategies in today’s markets: fundamentalists and chartists [7, 20]. Fundamentalists use a forecasting model that fits the actual economy and correctly identify the fundamental driving forces of the market. Technical analysts, also called chartists, use an autoregressive process to predict future price developments based on recent trends. In this paper we focus on fundamentalists, more specifically we look at the value of forecasting information by comparing the financial success of differently informed traders.

One might conjecture that more information is always better - if you know everything, you can act perfectly. However, previous work has shown that this does not need to be generally the case. In particular, it has been found both in simulation and in human experiments that averagely informed traders may be outperformed by uninformed traders that follow solely the current market price, and only insiders beat the market [13, 23]. One possible theory explaining this phenomenon is that more information helps during trends, whereas limited knowledge may be erroneous when the trend reverses; uninformed traders are safe from these systematic mistakes [10]. These findings were recently confirmed in simulation for larger numbers of traders and different distributions over traders’ information levels [9]. Moreover, an evolutionary analysis of the market showed that indeed only insiders can prevail when information is freely available. When information comes at a price, a mix of differently informed traders may be sustained in equilibrium [9].

Another factor that may greatly influence market outcomes is the presence of noise. Especially forecasting information may well be unreliable or imprecise, leading to a diminishing return on investment for information. In this paper we analyse the effect of noise by comparing different noise functions in a market with variously informed traders. Moreover, we compare the influence of noise and the influence of cost. Finally, combining both cost and noise gives rise to a tipping point where the acquisition of additional information costs more than it pays off.

The remainder of this paper is structured as follows. Section 2 briefly introduces auctions, evolutionary game theory and heuristic payoff tables, concepts necessary to understand the remainder of this work. The market model used in our experiments is detailed in Section 3. Experiments are described and results are presented in Section 4. Section 5 concludes.

2. BACKGROUND
This section provides relevant background needed for the remainder of this paper. Firstly, auctions and their application in stock markets are detailed. Secondly, evolutionary game theory and in particular the replicator dynamics are described. Finally, heuristic payoff tables, forming the core of our evolutionary analysis, are introduced.

2.1 Auctions
Auctions are highly efficient match making mechanisms for trading goods or services. As such, they are employed by a number of real markets, such as telecommunication spectrum rights auctions or the New York Stock Exchange (NYSE) [2, 15]. In practice, there is a variety of rules that may be used to conduct an auction. Each set of rules may result in different transaction volumes, transaction delays, or allocative market efficiency. One sided auctions, especially with one seller and many potential buyers, are popular in consumer-to-consumer e-commerce [3, 4]. Here, we focus on double auctions, which essentially provide a platform for buyers and sellers to meet and exchange a com-

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modality against money. A taxonomy of double auctions especially tailored to automated mechanism design can be found in [16].

Double auctions maintain an open book of bids (offers to buy at a specified price) and asks (offers to sell at a specified price). Two principle forms are the clearing house auction and continuous operation auction. In a clearing house auction, orders are collected for a trading period (e.g., one day) and matched, or cleared, after the trading period is closed. This mode of operation allows for higher allocative efficiency, but incurs delays in the transactions. In contrast, continuous operation immediately establishes a transaction as soon as some buyer is willing to pay more than a seller is asking for. This mode allows higher transaction rates at the cost of some allocative efficiency. Experiments in this article will use continuous operation mode, since it reflects the day-time operation mode of the NYSE [2].

2.2 Evolutionary game theory

Auctions provide a dynamic environment with a lot of traders (agents) that adapt to each other while competing for revenue. Learning in such multi-agent systems is generally complex and poses many challenges that inspire prescriptive, descriptive and normative research [19]. Evolutionary game theory provides a methodology to analyze multi-agent learning, replacing assumptions from game theory like rationality by evolutionary concepts such as pressure of natural selection [24].

The evolutionary perspective considers a population of individuals, where each individual belongs to one of several species. These species generally relate to atomic strategies, or to information levels within this article. Two core concepts are the replicator dynamics, describing how a population evolves, and evolutionarily stable states.

The replicator dynamics formally define the population change \( \dot{x} \) over time, where \( x \) describes the distribution of species in the population:

\[
\dot{x}_i = x_i \left[ f_i(x) - \sum_j x_j f_j(x) \right] \tag{1}
\]

The payoff function \( f_i(x) \) can be interpreted as the Darwinian fitness of each species \( i \). Intuitively, (1) describes how species that do better than average in the population thrive, whereas species that do worse decline. Evolutionarily stable state are such population distributions \( x \) that are fixed points of the replicator dynamics, i.e., \( \dot{x} = 0 \), and where small perturbations \( |\tilde{x} - x| < \epsilon \) would be driven back to \( x \) by selection pressure, i.e., by following the replicator dynamics.

Previous research has demonstrated the viability of evolutionary game theory to analyze meta strategies in simulated auctions, and to compare clearing house against continuous double auctions [12, 18]. We will follow a similar analysis procedure but our data is generated by a different model, described in Section 3.

2.3 Heuristic payoff tables

The evolutionary model assumes an infinite population. We cannot compute the payoff for such a population directly, but we can approximate it from evaluations of a finite population. All possible distributions over \( k \) information levels can be enumerated for a finite population of \( n \) individuals. Let \( N \) be a matrix, where each row \( N_i \) contains one discrete distribution. The matrix will yield \( \binom{n+k-1}{k} \) rows. Each distribution over information levels can be simulated using the market model described in Section 3, returning a vector of average expected relative market revenues \( u(N_i) \). Let \( U \) be a matrix which captures the revenues corresponding to the rows in \( N \), i.e., \( U_i = u(N_i) \). A heuristic payoff table \( H = (N, U) \) is proposed in [25] to capture the payoff information for all possible discrete distributions in a finite population.

In order to approximate the payoff for an arbitrary mix of strategies in an infinite population distributed over the species according to \( x \), \( n \) individuals are drawn randomly from the infinite distribution. The probability for selecting a specific row \( N_i \) can be computed from \( x \) and \( N_i \):

\[
P(N_i|x) = \binom{n}{N_{i,1}, N_{i,2}, \ldots, N_{i,k}} \prod_{j=1}^{k} x_j^{N_{i,j}}
\]

The expected payoff \( f(x) \) is computed as the weighted combination of the payoffs given in all rows:

\[
f_i(x) = \sum_j P(N_i|x) U_{j,i}
\]

If a discrete distribution features zero traders of a certain information type, its payoffs cannot be measured and \( U_{j,i} = 0 \). This expected payoff can be used in Equation 1 to compute the evolutionary population change according to the replicator dynamics.

3. Market model

The market is based on a continuous double auction with open order book, in which all traders can place bids and asks for shares. We closely follow the market model as described by [9, 11, 21] in order to be comparable. The intrinsic value of the shares is determined by a dividend stream that follows Brownian motion:

\[
D_t = D_{t-1} + \epsilon
\]

where \( D_t \) denotes the dividend in period \( t \) with \( D_0 = 0.2 \), and \( \epsilon \) is a normally distributed random term with \( \mu = 0 \) and \( \sigma = 0.01 \), i.e., \( \epsilon \sim \mathcal{N}(\mu, \sigma^2) \).

We simulate the market over 30 trading periods, each lasting 10-n time steps, where \( n \) is the number of traders present. All traders start with 1600 units cash and 40 shares, each worth 40 in the beginning. At the beginning of each period, all traders can put a bid or ask in the book (opening call). Hereafter, at every time step a trader is selected at random who can then either accept an open order, or place a bid or ask according to its trading strategy (see below). At the end of each period, dividend is paid based on the shares owned, and a risk free interest rate (0.1%) is paid over cash. The performance of the traders is measured as their total wealth after the 30 periods, i.e., each share is valued according to the discounted future dividends (see below) and added to the cash reserves.

3.1 Dividend Discount Model

Let us assume that we require a certain rate of return \( r > 0 \) on our investment. For example, if \( r = 0.005 \), a share must return 0.5% per trading period for it to be a worthwhile investment. Rate \( r \) is also called the discount rate. A future
dividend $D_t$ at time $t$ has a current discounted value of

$$\frac{D_t}{(1+r)^t}$$

If we intend to hold the share indefinitely, the value of a share is equal to the sum of future discounted dividends:

$$V = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}$$

Gordon’s growth model [8] assumes that dividends grow at a constant rate $g$. If $D_0$ is the current dividend payout, the current stock value can be computed as follows:

$$V = \sum_{t=1}^{\infty} D_0 \frac{(1+g)^t}{(1+r)^t} = D_0 \frac{1+g}{r-g}$$

Let us assume the dividends are constant over time, i.e. $\forall i: D_t = D$ and $g = 0$. The stock value simplifies to:

$$V = \sum_{t=1}^{\infty} \frac{D}{(1+r)^t}$$

(2)

The infinite series (2) converges to $\frac{D}{r}$ as $\frac{1}{1+g} < 0$ with $r > 0$. For example, a stock that pays a constant dividend of 0.2 per share has a current value of $V = 0.2/0.005 = 40$.

The different information levels are implemented by varying the amount of knowledge that traders have about future dividends. In trading period $t = k$, traders of information level $I_k$ know dividend $D_k$, and in general traders of information level $I_j$ know $D_k, \ldots, D_{k+j-1}$. Therefore, the discounted dividend payoff that is guaranteed with information level $I_j$ is

$$\sum_{i=0}^{j-1} \frac{D_{k+i}}{(1+r)^i}$$

and the future discounted dividends for $t > k + j - 1$ are estimated according to (2) with a constant $D = D_{k+j-1}$:

$$\sum_{t=k+j-1}^{\infty} D_{k+j-1} \frac{1}{(1+r)^t} = \frac{D_{k+j-1}}{r}$$

(3)

As (3) estimates future discounted dividends from period $t = k + j - 1$ on, (3) itself must be discounted by $\frac{1}{(1+r)^{k+j-1}}$ to adjust payouts to current value prices. The complete stock value estimate for information level $I_j$ is thus:

$$E(V|I_j, k) = \sum_{i=0}^{j-1} D_{k+i} \frac{1}{(1+r)^i} + \frac{D_{k+j-1}}{r(1+r)^{j-1}}$$

(4)

To put it intuitively, a trader of information level $I_j$ knows $j$ future dividends and assumes dividends stay fixed from that point on.

3.2 Trading strategies

We use two different trading strategies in our experiments. Fundamentals use their knowledge of future dividends ($I_0$ to $I_9$) to estimate the current value of the stock and base their trading decision on that estimate. Traders without any information ($I_0$) use the zero-information strategy that only makes use of the current market price of the shares.

Algorithm 1: Fundamentalist trading strategy

```java
pv ← E(V|I_j, k) according to Equation 4
if pv < bestBid then
    acceptOrder(bestBid)
else if pv > bestAsk then
    acceptOrder(bestAsk)
else
    if ∆ask > ∆bid then
        placeAsk(pv + 0.25 · ∆bid · N(0, 1))
    else
        placeBid(pv + 0.25 · ∆ask · N(0, 1))
end if
```

3.3 Cost and noise

Three types of cost function and noise are used in our experiments. In line with [9] we use a fixed cost function, where each fundamentalist pays the same fixed amount per trading period, and a quadratic cost function that is based
on the idea that it gets increasingly difficult to obtain more information. Whereas some forecasting information may be acquired by reading financial newspapers, a more detailed outlook may require hiring experts. Additionally we add a linear cost function, providing a middle ground between the former two. Figure 1 shows these three different cost functions.

Similarly, we employ three different types of noise functions to model uncertainty in forecasting data, depicted in Figure 2. Noise is added to each trader’s value estimate when executing Algorithm 1, drawn randomly from \( N(0, \sigma) \). In the case of fixed noise, each trader experiences the same level of uncertainty. More realistically, the uncertainty increases with the amount of forecasting, especially when e.g. step-by-step prediction is used [5]. This inspires the exponential noise function. Again, a linear function is used as well as compromise between these two.

4. RESULTS

This section details the experiments and lists their results. Two types of experiments are conducted to highlight the effect of both cost and noise on the relative return for differently informed traders. First, the population of traders is kept fixed, allowing us to visually investigate the market result when all information levels are present. Next, traders are allowed to switch their strategy if this is profitable. Evolutionary analysis of the resulting dynamical system indicates which strategy or information level is strongest from a natural selection point of view. Moreover, this analysis shows how the market evolves, and which strategy or strategies are economically viable in the long run under different scenarios.

4.1 Fixed market simulations

Previous work has shown that the value of information does not necessarily increase monotonically for traders in a stock market [13, 23]. Here we extend these results to situations in which information can be costly or subject to uncertainty. We simulate the market (see Section 3) with \( n = 100 \) traders, 10 for each of the information levels \( I_0, \ldots, I_9 \). Different scenarios are investigated using the various noise and cost functions described previously in Section 3.3. To reduce the effect of randomness we run 100 sessions of 100 simulations for each scenario; the dividend stream is fixed for each session. Results are given as the relative performance with respect to the market average plotted against the information levels.

Figure 3 shows the relative return over information level for different cost functions. When no cost is involved, the market behaves as expected based on previous work [13, 23]. Traders with little information are not able to make any profit, and would be better off not using their fundamental information at all. Averagely informed traders perform at market average, and only highly informed traders beat the market. Adding fixed cost or linear cost does not change the result dramatically and the J-curve is preserved, however zero information traders profit from not having to pay any cost. The situation changes when quadratic costs are incurred, and a tipping point can be observed when acquiring additional information costs more than it adds, as also reported in [9].

Adding noise has a similar effect, as is shown in Figure 4. Insiders still win under both fixed noise and linear noise, although by a smaller margin. Exponential noise changes the picture, again we observe a tipping point where the higher degree of uncertainty for longer forecasts outweighs their value. In other words, the signal-to-noise ratio of the forecasting information decreases with the length of forecasting.

Finally, we combine cost and noise in order to see how their effects might add up. Figure 5 shows the results. Where both linear cost and noise independently did not change the results dramatically, it is clear that their combined effect tips the scale: the relative return over information decreases monotonically. Interestingly, combining exponential noise and quadratic cost does not give rise to a monotonically decreasing relative return; instead averagely informed traders still profit enough from their fundamental information to make the added cost and uncertainty worthwhile.

It is clear from these results that both cost and noise can have a large influence on the relative performance of various trading strategies or information levels. So far, traders have not been able to choose their strategy, instead the distribu-
tion over information levels was kept fixed throughout the simulation. Realistically it can be assumed that traders may be inclined to switch strategy if they are currently performing poorly. In the next section we will look at the dynamics of a market in which traders are free to change their strategy.

4.2 Evolutionary analysis

We compute heuristic payoff tables, as described in Section 2.3, for various scenarios using different cost functions and different types of noise. Each heuristic payoff table is computed using market simulations with \( n = 12 \) traders of information levels \( I_0, I_1, \) and \( I_9 \), giving 91 rows in the payoff table for the various discrete distributions over these three types. We could have chosen different information levels, however previous work has indicated that this does not influence the results qualitatively [9]. For each row in the table we run the market simulation for 100 sessions of 10 simulations each. Again, the dividend stream is fixed for each session. As described in Section 2.3 we use the resulting payoff table to calculate expected fitnesses for an arbitrary mix of these strategies, and plug these in to the replicator dynamics (1) yielding a dynamical system that can be inspected visually.

Figure 6 shows the resulting market dynamics for different cost functions. In the absence of cost the best strategy is to get as much foresight as possible, leading to a domination of \( I_0 \) traders over the entire interior of the simplex. Adding fixed cost gives a small boost to the zero-information traders (as can also be observed in Figure 3), allowing them to survive in equilibrium alongside the insiders. Both linear and quadratic costs give rise to an internal equilibrium where each type of trader can survive.

Slightly different results are obtained in the case of noise and presented in Figure 7. Adding fixed noise does not change the evolutionary success of the insiders, and the pure \( I_0 \) equilibrium is still the main attractor. Linear noise shifts the equilibrium to a mix of insiders and zero-information traders; exponential noise leads to a situation where both averagely informed traders and insiders prevail - interestingly, in this scenario the zero-information traders are driven out of the market.

Finally, we again look at the situation where both cost and noise are present. Figure 8 shows the resulting dynamics. Each combination of cost and noise functions yields a mixed equilibrium where both zero-information and averagely informed traders prevail. The presence of both cost and noise tips the scale in favour of the less-informed, as acquiring more information becomes too costly to pay off.

4.3 Discussion

The various simulations and results show a variety of outcomes depending on whether or not cost and noise are a driving factor of the market. Whereas insiders drive all other traders out of the market when no noise is present and costs are zero, the situation rapidly changes and other types of traders can prevail when different types of noise or cost are added. This shows that the J-curve that was presented in previous work [9, 13, 23] may be not so much a general rule as it is a special case.

5. CONCLUSION

Previous work has established the non-trivial relation between the amount of forecasting information available to a trader and his or her expected return. Specifically, it has been observed that averagely informed traders perform below market average and are outperformed by zero-information traders - only insiders prevail. We extend previous work by investigating the effect of noise or uncertainty with respect to forecasting information. Realistically, a longer forecast horizon may lead to higher uncertainty or noise. We observe that the presence of noise changes the market outcome in such a way that averagely informed traders may coexist with insiders, driving zero-information traders out of the market. Putting a price on information similarly changes the market outcome, this time yielding an equilibrium where each type of trader can prevail simultaneously. Finally, a combination of cost and noise may lead to a situation where additional information is too costly to obtain, driving insiders out of the market.

As mentioned in the Introduction, real markets include not only fundamentalists, but technical analysts or chartists as well. A logical next step for future work is to include
Figure 6: Vector field showing the evolutionary dynamics of a market with three information levels and different cost functions for information. The noise is kept at zero.

Figure 7: Vector field showing the evolutionary dynamics of a market with three information levels and different types of noise over information. Cost for information is zero.
chairsts in the market model as well. Moreover, it would be of interest to investigate whether the price signal resulting from the market model in equilibrium exhibits stylised facts found in real financial time series data [1, 6, 14, 17], and if so under which circumstances. This would further validate our approach.

6. REFERENCES


